

# OPERATIONS RESEARCH

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## *Introduction to OR*

### *Terminology*

The British/Europeans refer to "operational research", the Americans to "operations research" - but both are often shortened to just "OR". Another term which is used for this field is "management science" ("MS"). The Americans sometimes combine the terms OR and MS together and say "OR/MS" or "ORMS". Yet other terms sometimes used are "industrial engineering" ("IE") and "decision science" ("DS"). In recent years there has been a move towards a standardisation upon a single term for the field, namely the term "OR".

In a nutshell, operations research (O.R.) is the discipline of applying advanced analytical methods to help make better decisions by using techniques such as mathematical modeling to analyze complex situations, operations research gives executives the power to make more effective decisions and build more productive systems based on:

- More complete data
- Consideration of all available options
- Careful predictions of outcomes and estimates of risk
- The latest decision tools and techniques

Thus, Operational Research (OR) is the use of advanced analytical techniques to improve decision making. It is sometimes known as Operations Research, Management Science or Industrial Engineering. People with skills in OR hold jobs in decision support, business analytics, marketing analysis and logistics planning – as well as jobs with OR in the title.

### **Why is OR needed?**

Because it makes sense to make the best use of available resources. Today's global markets and instant communications mean that customers expect high-quality products and services when they need them, where they need them. Organisations, whether public or private, need to provide these products and services as effectively and efficiently as possible. This requires careful planning and analysis – the hallmarks of good OR. This is usually based on process modelling, analysis of options or business analytics. O.R. is unique. It's best of breed, employing highly developed methods practiced by specially trained professionals. It's powerful, using advanced tools and technologies to provide analytical power that no ordinary software or spreadsheet can deliver out of the box.

To achieve these results, O.R. professionals draw upon the latest analytical technologies, including:

- **Simulation** Giving you the ability to try out approaches and test ideas for improvement
- **Optimization** Narrowing your choices to the very best when there are virtually innumerable feasible options and comparing them is difficult
- **Probability and Statistics** Helping you measure risk, mine data to find valuable connections and insights, test conclusions, and make reliable forecasts

**Operations research**, or **Operational Research** in British usage is a discipline that deals with the application of advanced analytical methods to help make better decisions. It is often considered to be a sub-field of Mathematics. The terms **management science** and **decision science** are sometimes used as more modern-sounding synonyms employing techniques from other mathematical sciences, such as mathematical modeling, statistical analysis, and mathematical optimization, operations research arrives at optimal or near-optimal solutions to complex decision-making problems. Because of its emphasis on human-technology interaction and because of its focus on practical applications, operations research has overlap with other disciplines, notably industrial engineering and operations management, and draws on psychology and organization science. Operations Research is often concerned with determining the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) of some real-world

objective. Originating in military efforts before World War II, its techniques have grown to concern problems in a variety of industries. Operational research (OR) encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency. Some of the tools used by operational researchers are statistics, optimization, probability theory, queuing theory, game theory, graph theory, decision analysis, mathematical modeling and simulation. Because of the computational nature of these fields, OR also has strong ties to computer science and analytics. Operational researchers faced with a new problem must determine which of these techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power.

Work in operational research and management science may be characterized as one of three categories:

1. Fundamental or foundational work takes place in three mathematical disciplines: probability theory, mathematical optimization, and dynamical systems theory. 2. Modeling work is concerned with the construction of models, analysing them mathematically, implementing them on computers, solving them using software tools, and assessing their effectiveness with data. This level is mainly instrumental, and driven mainly by statistics and econometrics.

3. Application work in operational research, like other engineering and economics disciplines, attempts to use models to make a practical impact on real-world problems.

The major sub disciplines in modern operational research are:

- Computing and information technologies
- Decision analysis
- Environment, energy, and natural resources
- Financial engineering
- Manufacturing, service sciences, and supply chain management
- Marketing Engineering
- Policy modeling and public sector work
- Revenue management
- Simulation
- Stochastic models

## **History of OR**

As a formal discipline, operational research originated in the efforts of military planners during World War II. In the decades after the war, the techniques began to be applied more widely to problems in business, industry and society. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyse and optimize complex systems, and has become an area of active academic and industrial research.

In the World War II era, operational research was defined as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control." Other names for it included operational analysis (UK Ministry of Defence from 1962) and quantitative management.

Prior to the formal start of the field, early work in operational research was carried out by individuals such as Charles Babbage. His research into the cost of transportation and sorting of mail led to England's universal "Penny Post" in 1840, and studies into the dynamical behaviour of railway vehicles in defence of the GWR's broad gauge. Percy Bridgman brought operational research to bear on problems in physics in the 1920s and would later attempt to extend these to the social sciences. The modern field of operational research arose during World War II.

Modern operational research originated at the Bawdsey Research Station in the UK in 1937. A. P. Rowe conceived the idea as a means to analyse and improve the working of the UK's early warning radar system, Chain Home (CH).

Scientists in the United Kingdom including Patrick Blackett (later Lord Blackett OM PRS), Cecil Gordon, C. H. Waddington, Owen Wansbrough-Jones, Frank Yates, Jacob Bronowski and Freeman Dyson, and in the United States with George Dantzig looked for ways to make better decisions in such areas as logistics and training schedules. After the war it began to be applied to similar problems in industry.

## **Second World War**

During the Second World War close to 1,000 men and women in Britain were engaged in operational research. About 200 operational research scientists worked for

the British Army. Patrick Blackett worked for several different organizations during the war. Early in the war while working for the Royal Aircraft Establishment (RAE) he set up a team known as the "Circus" which helped to reduce the number of anti-aircraft artillery rounds needed to shoot down an enemy aircrafts.

## **After World War II**

With expanded techniques and growing awareness of the field at the close of the war, operational research was no longer limited to only operational, but was extended to encompass equipment procurement, training, logistics and infrastructure.

## **Problems addressed with Operational Research (Scope of OR)**

1. Critical path analysis or project planning: identifying those processes in a complex project which affect the overall duration of the project
2. Floor planning: designing the layout of equipment in a factory or components on a computer chip to reduce manufacturing time (therefore reducing cost)
3. Network optimization: for instance, setup of telecommunications networks to maintain quality of service during outages
4. Allocation problems
5. Facility location
6. Assignment Problems:
7. Assignment problem
8. Generalized assignment problem
9. Quadratic assignment problem
10. Weapon target assignment problem
11. Bayesian search theory : looking for a target
12. Optimal search
13. Routing, such as determining the routes of buses so that as few buses are needed as possible

14. Supply chain management: managing the flow of raw materials and products based on uncertain demand for the finished products

15. Efficient messaging and customer response tactics

16. Automation: automating or integrating robotic systems in human-driven operations processes

17. Globalization: globalizing operations processes in order to take advantage of cheaper materials, labor, land or other productivity inputs

18. Transportation: managing freight transportation and delivery systems (Examples: LTL Shipping, intermodal freight transport)

19. Scheduling:

20. Personnel staffing

21. Manufacturing steps

22. Project tasks

23. Network data traffic: these are known as queueing models or queueing systems.

24. Sports events and their television coverage

25. Blending of raw materials in oil refineries

26. Determining optimal prices, in many retail and B2B settings, within the disciplines of pricing science

27. Operational research is also used extensively in government where evidence-based policy is used.

### **Management Science**

In 1967 Stafford Beer characterized the field of management science as "the business use of operations research". However, in modern times the term management science may also be used to refer to the separate fields of organizational studies or corporate strategy. Like operational research itself, management science (MS) is an interdisciplinary branch of applied mathematics devoted to optimal decision planning. It uses various scientific research-based principles to arrive at optimal or near

optimal solutions to complex decision problems. In short, management sciences help businesses to achieve their goals using the scientific methods of operational research.

### **Applications of management science**

Applications of management science are abundant in industry as airlines, manufacturing companies, service organizations, military branches, and in government. The range of problems and issues to which management science has contributed insights and solutions is vast. It includes:

- scheduling airlines, including both planes and crew,
- deciding the appropriate place to site new facilities such as a warehouse, factory or fire station,
- managing the flow of water from reservoirs,
- identifying possible future development paths for parts of the telecommunications industry,
- establishing the information needs and appropriate systems to supply them within the health service, and
- identifying and understanding the strategies adopted by companies for their information systems

### DEFINITIONS

On an historical note the Encyclopedia Britannica notes that the word algorithm derives from the Latin translation, *Algoritmi de numero Indorum*, of the 9th-century Muslim mathematician Abu Ja'far Muhammad ibn Musa Al-Khwarizmi who wrote "*Al-Khwarizmi Concerning the Hindu Art of Reckoning*."

1. OR is the representation of real-world systems by mathematical models together with the use of quantitative methods (algorithms) for solving such models, with a view to optimising.

2. O.R. is the art of winning wars without actually fighting – Arthur Clarke

3. O.R. is the art of winning wars without actually fighting. - Arthur Clarke
4. O.R. is concerned with scientifically deciding how to best design and operate man-machine systems usually under conditions requiring the allocation of scarce resources. - O.R. Society of America
5. O.R. is the art of giving bad answers to problems which otherwise have worse answers. -T.L. Saaty
- 6.. O.R. is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problems. -D.W. Miller and M.K. Starr
- 7.. O.R. is a scientific approach to problems solving for executive management. -H.M. Wagner
8. O.R. is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solution to the problem. -Churchman, Ackoff and Arnoff
9. O.R. is the study of administrative system pursued in the same scientific manner in which systems in Physics, Chemistry and Biology are studied in natural sciences.
- 10.. O.R. is scientific methodology-analytical, experimental, quantitative-which by assessing the overall implication of various alternative courses of action in a management system, provides an improved basis for management decisions. –Pocock
- 11.. O.R. is the application of the theories of Probability, Statistics, Queuing, Games, Linear Programming, etc. to the problems of war, govt. and industry.
12. O.R. is the use of scientific methods to provide criteria for decisions regarding man machine systems involving repetitive operations.

So mathematical model consists of:

- Decision *variables*, which are the unknowns to be determined by the solution to the model.
- *Constraints* to represent the physical limitations of the system.
- An *objective* function.
- A *solution* (or *optimal solution*) to the model is the identification of a set of variable values which are feasible (i.e. satisfy all the constraints) and which lead to the optimal value of the objective function.

## **Modeling in Operations Research**

A model in Operations Research is defined as an idealised representation of the real life situation. A model is the representation of an actual object or situation. Another definition of model is given as “Models are abstractions built to understand a problem before implementing a solution .” Model shows the relationship and interrelationship of an action and reaction in terms of cause and effect.

Since a model is an abstraction of the reality, it appears to be less complete than the reality itself. For a model to be complete, it must be representative of those aspects of reality that are being investigated. The main objective of a model is to provide means for analysing the behaviour of the system for the purpose of improving its performance. The reliability of the solution is obtained from model depends on the validity of the model in representing the real system. A model permits to examine the behaviour of the system without interfering with ongoing operations.

### **Classification of Models**

Models can be classified according to the following characteristics.

#### **A .Classification by Structure**

##### i) ICONIC Models or Physical Models

In ICONIC or Physical Models, the properties of the real system are represented by the properties themselves, frequently with change of scales. Thus ICONIC models resemble the system they represent but differ in sizes. They are images. For example, globes are used to represent the orientation and shape of various continents and other

geographical features of earth. A model of solar system, a toy aeroplane are iconic models of real ones.

#### ii)Analogue Model:

The models, in which one set of properties is used to represent another set of properties, are called analogue models. After the solution is obtained, it is reinterpreted in terms of the original system. For example, graphs are simple analogues because distance is used to represent the properties such as time, number, percent, age, weight and many other properties.

#### iii)Symbolic Model:

Symbolic models employ a set of mathematical symbols, letters, numbers etc. to represent the decision variables of the system under study. These variables are related together by mathematical equations or inequalities which describe the properties of the system. A solution from the model is obtained by applying well developed mathematical techniques.

### **B .Classification by Purpose**

Models can be classified by the purpose of its utility as explained below

#### i)Descriptive Model:

It is the model which simply describes some aspects of a situation based on observations,survey, questionnaire results or other available data. The result of an opinion poll represents a descriptive model.

#### ii)Predictive Models:

Such models can answer ‘what if ’ types of questions i.e. they can make predictions regarding certain events. For example based on survey results, television networks such models attempt to explain and predict the election results before all the votes are actually counted.

#### iii)Prescriptive Models:

They are prescribing the source of action. For example Linear Programming is a prescriptive model because it prescribes what the managers ought to do.

### **C. Classification by Nature of Environment:**

#### i) Deterministic Model:

In this model variables are completely defined and the outcomes are certain. Certainty is the state of nature assumed in these models. They represent a completely closed system and the results are single valued. For any given set of input variables, the same set of output variables always result. For example Linear Programming, transportation and assignment problems, Economic Order Quantity models are deterministic models.

#### ii) Probabilistic or Stochastic Models:

They usually handle situations in which the consequences or payoffs of managerial actions cannot be predicted with certainty. They are semi closed models and represent the likelihood of occurrence of an event. However, it is possible to forecast a pattern of events, based on which managerial decisions can be made. For example the insurance companies are willing to insure against risk of fire, accidents, sickness and so on, because the pattern of events have been compiled in the form of probability distributions.

### **D. Classification by Behaviour/Time Horizon**

#### i) Static Models

These models do not consider the impact of changes that take place during the planning horizon i.e. they are independent of time. In this model only one decision is needed for the duration of a given time period.

#### ii) Dynamic Models:

In these models, time is considered as one of the important variables and admit the impact of changes generated by time. They are used for optimisation of multi stage

decisions problems which require a series of decisions with the outcome of each depending upon the results of the previous decisions in the series.

### **E. Classification by Method of Solution:**

#### i)Analytical Model:

These models have a specific mathematical structure and thus each can be solved by known analytical or mathematical techniques. For example, a general Linear Programming, transportation and assignment models.

#### ii)Simulation Models:

They also have mathematical structure but they cannot be solved by purely using the tools of and techniques of mathematics. A simulation model is essentially a computer assisted experimentation on mathematical structure of a real time structure in order to study the system under a variety of assumptions.

### **F. Classification on the Basis of Generality:**

#### i)General Models:

General models are those models that can be used for all functions. For example LPP models used for functions such as product mix, productions scheduling, marketing etc.

#### ii)Specific Models:

They are used for a specific purpose. Example: sales response curve or equation as a function of advertising is applicable in the marketing function alone.

### **G. Classification on the Basis of Functions Performed**

#### i) Function Models:

Such models are generally grouped on the basis of functions being performed. Examples: tables carrying data, a blue print of lay outs, a programme representing a sequence of operations like computer programming.

## ii)Quantitative Models:

They are used to measure the observations. Example: Degree of temperature, yardstick, a unit of measurement of length value.

## iii)Heuristic Models:

These models are mainly used to explore the alternative strategies (courses of action) that were overlooked previously. They do not claim to find the best situation to the problem.

### *Phases of an OR project*

Drawing on our experience with the Two Mines problem we can identify the phases that a (real-world) OR project might go through.

#### **1. Problem identification**

- Diagnosis of the problem from its symptoms if not obvious (i.e. what is the problem?)
- Delineation of the subproblem to be studied. Often we have to ignore parts of the entire problem.
- Establishment of objectives, limitations and requirements.

#### **2. Formulation as a mathematical model**

It may be that a problem can be modelled in differing ways, and the choice of the appropriate model may be crucial to the success of the OR project. In addition to algorithmic considerations for solving the model (i.e. can we solve our model numerically?) we must also consider the availability and accuracy of the real-world data that is required as input to the model.

Note that the "**data barrier**" ("we don't have the data!!!") can appear here, particularly if people are trying to block the project. Often data can be collected/estimated, particularly if the potential benefits from the project are large enough.

You will also find, if you do much OR in the real-world, that some environments are naturally *data-poor*, that is the data is of poor quality or nonexistent and some

environments are naturally *data-rich*. As examples of this I have worked on a church location study (a data-poor environment) and an airport terminal check-in desk allocation study (a data-rich environment).

This issue of the data environment can affect the model that you build. If you believe that certain data can never (realistically) be obtained there is perhaps little point in building a model that uses such data.

### **3. Model validation (or algorithm validation)**

Model validation involves running the algorithm for the model on the computer in order to ensure:

- the input data is free from errors
- the computer program is bug-free (or at least there are no outstanding bugs)
- the computer program correctly represents the model we are attempting to validate
- the results from the algorithm seem reasonable (or if they are surprising we can at least understand why they are surprising). Sometimes we feed the algorithm historical input data (if it is available and is relevant) and compare the output with the historical result.

### **4. Solution of the model**

Standard computer packages, or specially developed algorithms, can be used to solve the model (as mentioned above). In practice, a "solution" often involves very many solutions under varying assumptions to establish sensitivity. For example, what if we vary the input data (which will be inaccurate anyway), then how will this effect the values of the decision variables? Questions of this type are commonly known as "what if" questions nowadays.

Note here that the factors which allow such questions to be asked and answered are:

- the speed of processing (turn-around time) available by using pc's; and

- the interactive/user-friendly nature of many pc software packages.

## **5. Implementation**

This phase may involve the implementation of the results of the study or the implementation of the *algorithm* for solving the model as an operational tool (usually in a computer package). In the first instance detailed instructions on what has to be done (including time schedules) to implement the results must be issued. In the second instance operating manuals and training schemes will have to be produced for the effective use of the algorithm as an operational tool.

It is believed that many of the OR projects which successfully pass through the first four phases given above fail at the implementation stage (i.e. the work that has been done does not have a lasting effect). As a result one topic that has received attention in terms of bringing an OR project to a successful conclusion (in terms of implementation) is the issue of *client involvement*. By this is meant keeping the client (the sponsor/originator of the project) informed and consulted during the course of the project so that they come to identify with the project and want it to succeed. Achieving this is really a matter of experience.

### **Phases and Processes of O.R.**

#### **Formulate the problem:**

This is the most important process, it is generally lengthy and time consuming. The activities that constitute this step are visits, observations, research, etc. With the help of such activities, the O.R. scientist gets sufficient information and support to proceed and is better prepared to formulate the problem. This process starts with understanding of the organizational climate, its objectives and expectations. Further, the alternative courses of action are discovered in this step.

#### **Develop a model:**

Once a problem is formulated, the next step is to express the problem into a mathematical model that represents systems, processes or environment in the form of

equations, relationships or formulas. We have to identify both the static and dynamic structural elements, and device mathematical formulas to represent the interrelationships among elements. The proposed model may be field tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied with the answer that it gives.

### **Select appropriate data input:**

Garbage in and garbage out is a famous saying. No model will work appropriately if data input is not appropriate. The purpose of this step is to have sufficient input to operate and test the model.

### **Solution of the model:**

After selecting the appropriate data input, the next step is to find a solution. If the model is not behaving properly, then updating and modification is considered at this stage.

### **Validation of the model:**

A model is said to be valid if it can provide a reliable prediction of the system's performance. A model must be applicable for a longer time and can be updated from time to time taking into consideration the past, present and future aspects of the problem.

### **Implement the solution:**

The implementation of the solution involves so many behavioural issues and the implementing authority is responsible for resolving these issues. The gap between one who provides a solution and one who wishes to use it should be eliminated. To achieve this, O.R. scientist as well as management should play a positive role. A properly implemented solution obtained through O.R. techniques results in improved working and wins the management support.

## **Advantages & Limitations of Operations Research**

### **Advantages**

#### **1. Better Control:**

The management of large organizations recognize that it is a difficult and costly affair to provide continuous executive supervision to every routine work. An O.R. approach may provide the executive with an analytical and quantitative basis to identify the problem area. The most frequently adopted applications in this category deal with production scheduling and inventory replenishment.

### **2. Better Systems:**

Often, an O.R. approach is initiated to analyze a particular problem of decision making such as best location for factories, whether to open a new warehouse, etc. It also helps in selecting economical means of transportation, jobs sequencing, production scheduling, replacement of old machinery, etc.

### **3. Better Decisions:**

O.R. models help in improved decision making and reduce the risk of making erroneous decisions. O.R. approach gives the executive an improved insight into how he makes his decisions.

### **4. Better Co-ordination:**

An operations-research-oriented planning model helps in co-ordinating different divisions of a company.

## **Limitations**

### **1. Dependence on an Electronic Computer:**

O.R. techniques try to find out an optimal solution taking into account all the factors. In the modern society, these factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations that can only be handled by computers.

### **2. Non-Quantifiable Factors:**

O.R. techniques provide a solution only when all the elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors that cannot be quantified find no place in O.R. models.

### **3.Distance between Manager and Operations Researcher:**

O.R. being specialist's job requires a mathematician or a statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of O.R. Thus, there is a gap between the two.

### **4.Money and Time Costs:**

When the basic data are subjected to frequent changes, incorporating them into the O.R. models is a costly affair. Moreover, a fairly good solution at present may be more desirable than a perfect O.R. solution available after sometime.

### **5.Implementation:**

Implementation of decisions is a delicate task. It must take into account the complexities of human relations and behaviour.

**Some problems that can be analyzed by operations research approach are classified as follows:**

#### 1. Finance, Budgeting and Investments

Credit policy analysis.

Cash flow analysis.

Dividend policies.

Investment portfolios.

#### 2. Marketing

Product selection, timing, etc.

Advertising media, budget allocation.

Number of salesman required.

Selection of product mix.

### 3. Purchasing, Procurement and Exploration

Optimal buying and reordering.

Replacement policies

### 4. Production Management

Location and size of warehouses, factories, retail outlets, etc.

Distribution policy.

Loading and unloading facilities for trucks, etc.

Production scheduling.

Optimum product mix.

Project scheduling and allocation of resources.

### 5. Personnel Management

Selection of suitable personnel.

Recruitment of employees.

Assignment of jobs.

Skills balancing.

### 6. Research and Development

Project selection.

Control of R&D projects.

Reliability and alternative design.